

The Physics of Music

When a stringed instrument is plucked, the resulting sound is comprised of the pitch of the string (the fundamental) plus quieter harmonic vibrations of the string as shown in the example on the right. This is known as the overtone series.

Fundamental	Freq. Hz	Note	
1	132	C	
Harmonic			
2	264	C2	One Octave Higher
3	396	G	5th above C2
4	528	C3	Two octaves above fundamental
5	660	E	3rd above C3
6	792	G	5th above C3

Just Tuning

The Just Scale (sometimes referred to as harmonic tuning) occurs naturally as a result of the overtone series for simple systems such as vibrating strings. All the notes in the scale are related by rational numbers. Unfortunately, with Just tuning, the tuning depends on the scale you are using - the tuning for C Major is not the same as for D Major, for example.

The steps of the scale are found by using these simple fractions of the root frequency:

1st	2nd	3rd	4th	5th	6th	7th	octave
1	9/8	5/4	4/3	3/2	5/3	15/8	2/1

Here are the note frequencies using the formula above. Note the pitches in bold are in conflict with the same notes from other scales.

C 264	D 297	E 330	F 352	G 396	A 440	B 495	C 528
G 396	A 445.5	B 495	C 528	D 594	E 660	F# 742.5	G 792
D 297	E 334.1	F# 371.2	G 396	A 445.5	B 495	C# 556.9	D 594
F 352	G 396	A 440	Bb 469.3	C 528	D 586.7	E 660	F 704

Equal Tempered Tuning

For the equal tempered scale, the frequency of each succeeding note in the chromatic scale is calculated by multiplying the current note by the twelfth root of 2 (1.059463...). All keys use the same pitches.

A	220.00 x 1.059 = 233.08 = Bb
Bb	233.08 x 1.059 = 246.94 = B
B	246.94 x 1.059 = 261.62 = C
C	261.63 x 1.059 = 277.18 = C#
C#	277.18 x 1.059 = 293.66 = D
D	293.66 x 1.059 = 311.12 = D#
D#	311.13 x 1.059 = 329.62 = E
E	329.63 x 1.059 = 349.23 = F
F	349.23 x 1.059 = 369.99 = F#
F#	369.99 x 1.059 = 391.99 = G
G	392.00 x 1.059 = 415.30 = G#
G#	415.30 x 1.059 = 440.00 = A
A	440.00

Notes of a G chord

Just tuning:

G	196.00
B	245.00
D	294.00

Equal tempered tuning:

G	196.00	Out of tune by:
B	246.94	1.94
D	293.66	-0.34